TEMPERATURE DISTRIBUTION IN A SHOCK-HEATED REINFORCED PANEL

A. V. Samarin, B. M. Klimenko,
R. E. Liberzon, and B. P. Rumyantsev

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The temperature distribution has been determined for a rod bearing a localized mass; the solution has been obtained as rapidly convergent series for small and large times. Experimental results are given.

Consider the temperature distribution in a panel with reinforcing elements (stringers, ribs, corrugations, and so on), which is subject to a shock heat load. We neglect any contact thermal resistance between the panel and the reinforcing elements. We also assume that the size of the area of contact is small by comparison with the size of the panel. In that case, the determination of the temperature distribution can be reduced to the heating of a rod of length l bearing a localized mass m at the end (Fig. 1).

The solution will be derived for the case where the heat pulse is approximated by bilinear dependence of q on τ (Fig. 2):

$$q(\tau) = K_1\tau - \eta(\tau - \tau_1)(K_1 - K_2)(\tau - \tau_1) + \eta(\tau - \tau_2)(-K_2)(\tau - \tau_2), \tag{1}$$

where $\eta(\tau - \tau_1)$ is a Heaviside function.

We make the following assumptions which simplify the problem but which do not introduce substantial errors into the final results:

1) the rod is thin (we neglect the nonuniform temperature distribution in the thickness of the plate);

2) the temperature of a reinforcing element is dependent only on time; and

3) the thermophysical characteristics of the plate and reinforcing-element materials are independent of temperature.

We have to determine the temperature of the rod $t_1(x, \tau)$ and of the reinforcing element $t_2(\tau)$ to satisfy the equation

$$c\gamma \frac{\partial t_1}{\partial \tau} = \lambda \frac{\partial^2 t_1}{\partial x^2} + \frac{q(\tau)}{h}, \tag{2}$$

$$\lambda_b \frac{\partial t_1}{\partial x}\bigg|_{x=0} = cm \frac{\partial^2 t}{\partial \tau}, \tag{3}$$

and the following initial conditions ($\tau = 0$):

$$t_1 - t_0 = 0, \; t_2 - t_0 = 0, \tag{4}$$

the boundary conditions ($x = l$)

$$\frac{\partial t_1(l, x)}{\partial x} = 0 \tag{5}$$

and the linkage conditions ($x = 0$):

$$t_1(0, \tau) = t_2(\tau). \tag{6}$$

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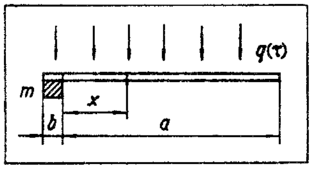

Fig. 1. The panel.

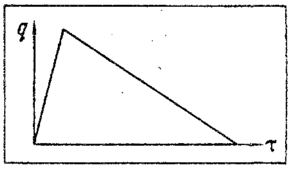

Fig. 2. The $q(\tau)$ curve.

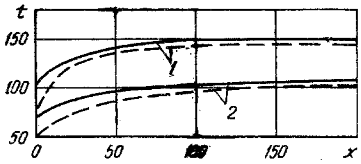

Fig. 3. Temperature distribution in panel; solid lines from calculation, broken lines from measurements: 1) τ = 3 sec; 2) 1 sec; t is in °C; x is in mm.

The boundary-value problem of (2)-(6) is solved by Laplace transformation; the original for the temperature distribution is in the form

$$c\gamma h t(\bar{x}, \tau) = q(\tau) - \frac{1}{1+\bar{H}} \int_0^\tau q(\tau-\xi)\, d\xi - \sum_{n=1}^{\infty} \int_0^\tau q(\tau-\xi) \frac{\cos \mu_n (1-\bar{x})}{\left[\frac{3}{2} + \frac{\bar{H}}{2} + \frac{\mu_n^2}{2\bar{H}}\right] \cos \mu_n} e^{-\mu_n \xi}\, d\xi, \tag{7}$$

where

$$\bar{H} = H\frac{l}{\sqrt{a}}, \quad \bar{x} = \frac{x}{l}, \quad H = \frac{\lambda_b}{cm\sqrt{a}}, \quad a = \frac{\lambda}{c\gamma},$$

and μ_n are the roots of the characteristic equation

$$\operatorname{tg} \mu = -\frac{\mu}{\bar{H}}. \tag{8}$$

In a rapid process, such as thermal shock, the maximal thermal stresses arise at small times τ; the following is the asymptotic solution to (2)-(6) for small τ on the basis of (1):

$$c\gamma t(\tau, x) = \int_0^\tau q_1(\tau)\, d\tau - K_1 \sum_{n=0}^{\infty} (-H)^n \Big\{ (4\tau)^{\frac{4+n}{2}} i^{(4+n)} \times \operatorname{erfc}\frac{x}{2\sqrt{a\tau}} - \frac{\tau_2}{\tau_2-\tau_1} [4(\tau-\tau_1)]^{\frac{4+n}{2}} i^{4+n} \operatorname{erfc}\frac{x}{2\sqrt{a(\tau-\tau_1)}} + \frac{\tau_1}{\tau_2-\tau_1} [4(\tau-\tau_2)]^{\frac{4+n}{2}} i^{4+n} \operatorname{erfc}\frac{x}{2\sqrt{a(\tau-\tau_2)}} \Big\}. \tag{9}$$

Calculations show that sufficient accuracy is obtained by retaining the first two terms in the latter expression.

The temperature distribution in a reinforcing element is readily derived from (9) as

$$hc\gamma t_2(\tau) = \int_0^\tau q_1(\tau)\, d\tau - K_1 \sum_{n=0}^{\infty} (-1)^n H^n \Big\{ (4\tau)^{\frac{4+n}{2}} \frac{1}{2^{4+n}\Gamma\left(1+\frac{4+n}{2}\right)} - \frac{\tau_2}{\tau_2-\tau_1} [4(\tau-\tau_1)]^{\frac{4+n}{2}} \frac{\eta(\tau-\tau_1)}{2^{4+n}\Gamma\left(1+\frac{4+n}{2}\right)} + \frac{\tau_1}{\tau_2-\tau_1} [4(\tau-\tau_2)]^{\frac{4+n}{2}} \frac{\eta(\tau-\tau_2)}{2^{4+n}\Gamma\left(1+\frac{4+n}{2}\right)} \Big\}. \tag{10}$$

The results are given below for a steel plate with the following data: λ = 0.068 kW/m·deg, b = 0.003 m; c = 0.445 kW·sec/kg·deg, m = 2.73 kg/η, K_2 = 17.5 kW/m^2·sec, a = 0.215·10^{-4} m^2/sec, H = λ_b/cm $\sqrt{a}$ = 0.375 1/sec$^{1/2}$, K_1 = 100 kW/m^2·sec, τ_1 = 1 sec, τ = 1 sec, 3 sec.

We retained four terms in the series using (9); Fig. 3 shows the results for the temperature of the plate as a function of coordinates and time.

These working formulas were checked by comparing the calculations with measurements on shock heating for a reinforced plate; we used an infrared-heating system fitted with quartz-lamp heaters KG × 220 × 2500-3, which provided a heat flux at the specimen of 700 kW/m^2. A thermal pulse was provided by a fast shutter which cut off the heat flux while the lamps were heating up, and its opening time thus controlled the rise time of the pulse. The trailing edge of the pulse was provided by a thyristor regulator, which ran the voltage on the lamps down to zero in a set fashion.

The specimen was a steel plate 600 × 600 mm of thickness 1 mm with reinforcing elements welded around the edge.

Figure 3 shows the results.

NOTATION

q, heat flux; τ, time; t, temperature; c, specific heat of plate material and reinforcing element; γ, density of panel material; m, linear density of reinforcing material; λ, thermal conductivity of plate material.

LITERATURE CITED

1. A. V. Lykov, Theory of Thermal Conductivity [in Russian], Nauka, Moscow (1967).